MATHEMATICS
MASTERY

Calculations policy
September 2014

Introduction

At the centre of the mastery approach to the teaching of mathematics is the belief that all children have the potential to succeed. They should have access to the same curriculum content and, rather than being extended with new learning, they should deepen their conceptual understanding by tackling challenging and varied problems. Similarly with calculation strategies, children must not simply rote learn procedures but demonstrate their understanding of these procedures through the use of concrete materials and pictorial representations.

This policy outlines the different calculation strategies that should be taught and used in Year 1 and Year 2 in line with the requirements of the 2014 Primary National Curriculum.

Background

The 2014 Primary National Curriculum for mathematics differs from its predecessor in many ways. Alongside the end of Key Stage year expectations, there are suggested goals for each year; there is also an emphasis on depth before breadth and a greater expectation of what children should achieve. In addition, there is a whole new assessment method, as the removal of levels gives schools greater freedom to develop and use their own systems.

One of the key differences is the level of detail included, indicating what children should be learning and when. This is suggested content for each year group, but schools have been given autonomy to introduce content earlier or later, with the expectation that by the end of each key stage the required content has been covered.

For example, in Year 2, it is suggested that children should be able to 'add and subtract one-digit and two-digit numbers to 20, including zero' and a few years later, in Year 5, they should be able to 'add and subtract whole numbers with more than four digits, including using formal written methods (columnar addition and subtraction)'.

In many ways, these specific objectives make it easier for teachers to plan a coherent approach to the development of pupils' calculation skills. However, the expectation of using formal methods is rightly coupled with the explicit requirement for children to use concrete materials and create pictorial representations – a key component of the mastery approach.

Purpose

The purpose of this policy is twofold. Firstly, it makes teachers aware of the strategies that pupils are formally taught within each year group that will support them to perform mental and written calculations. Secondly, it supports teachers in identifying appropriate pictorial representations and concrete materials to help develop understanding.

The policy only details the strategies; teachers must plan opportunities for pupils to apply these; for example, when solving problems, or where opportunities emerge elsewhere in the curriculum.

How to use the policy

For ease of reference, the strategies and examples contained in this policy are cross-referenced with objectives from the *2014 Maths Programme of Study*. For each of the four rules of number, different strategies are laid out, together with examples of what concrete materials can be used and how, along with suggested pictorial representations. Please note that the concrete and representation examples are not exhaustive, and teachers and pupils may well come up with alternatives. Where necessary, additional guidance is given to support in teaching the given strategies.

Please note that the principle of the concrete-pictorial-abstract (CPA) approach is that for children to have a true understanding of a mathematical concept, they need to master all three phases. Reinforcement is achieved by going back and forth between these representations. For example, if a child has moved on from the concrete to the pictorial, it does not mean that the concrete cannot be used alongside the pictorial. Or if a child is working in the abstract, 'proving' something or 'working out' could involve use of the concrete or pictorial. In short, these are not always 'exclusive' representations.

Mathematical language

The 2014 National Curriculum is explicit in articulating the importance of children using the correct mathematical language as a central part of their learning. Indeed, in certain year groups, the non-statutory guidance highlights the requirement for children to extend their language around certain concepts.

It is therefore essential that teaching using the strategies outlined in this policy is accompanied by the use of appropriate mathematical vocabulary. New vocabulary should be introduced in a suitable context (for example, with relevant real objects, apparatus, pictures or diagrams) and explained carefully.

> The quality and variety of language that pupils hear and speak are key factors in developing their mathematical vocabulary and presenting a mathematical justification, argument or proof.
>
> *2014 Maths Programme of Study*

High expectations of the mathematical language used are essential, with teachers only accepting what is correct.

✓	×
ones	units
is equal to	equals
zero	oh (the letter O)

MATHEMATICS MASTERY

Progression in calculations

Year 1

AN ark PROGRAMME

MATHEMATICS MASTERY

National Curriculum objectives linked to addition and subtraction

These objectives are explicitly covered through the strategies outlined in this document:

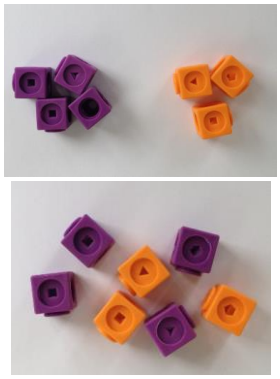
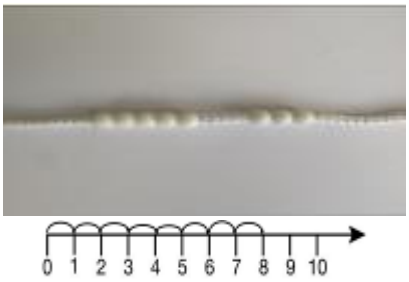
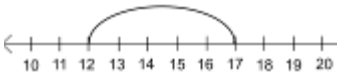
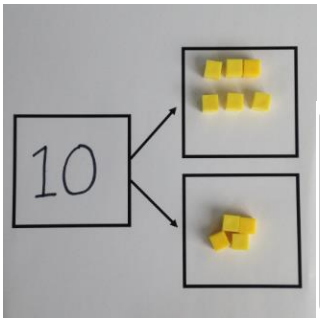
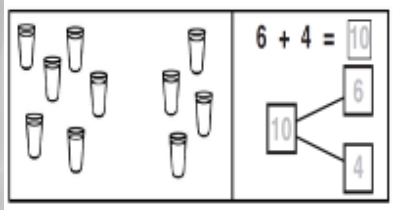
- Add and subtract one-digit and two-digit numbers to 20, including zero (Year 1).
- Add and subtract numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and ones, a two-digit number and tens, 2 two-digit numbers; add 3 one-digit numbers (Year 2).
- Show that addition of two numbers can be done in any order (commutative) but subtraction of one number from another cannot (Year 2).
- Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.

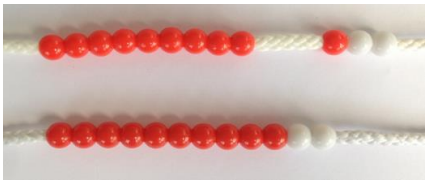
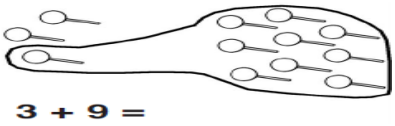
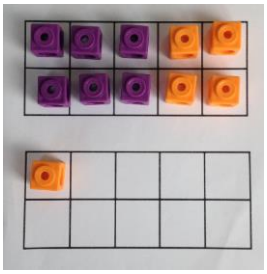
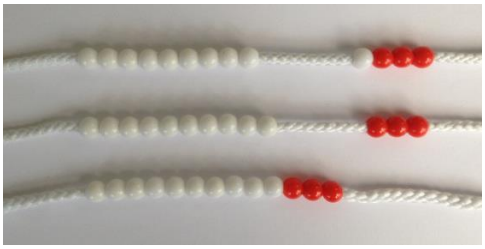
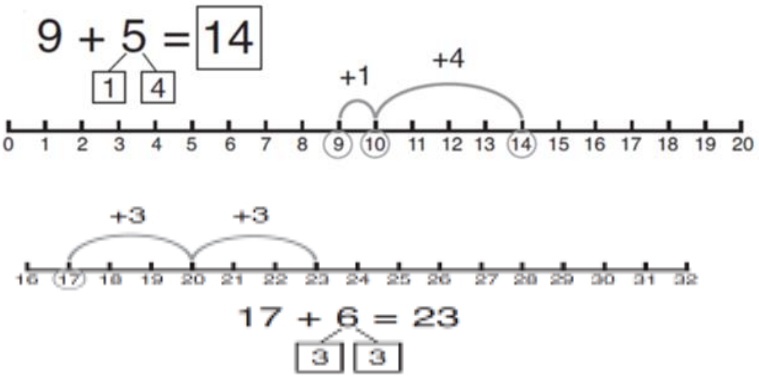
The following objectives should be planned for lessons where new strategies are being introduced and developed:

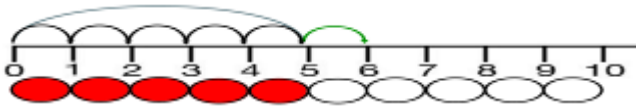
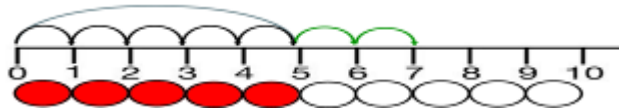
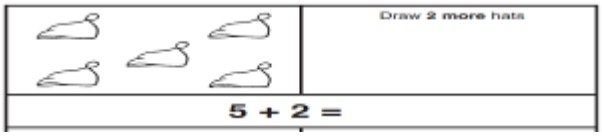
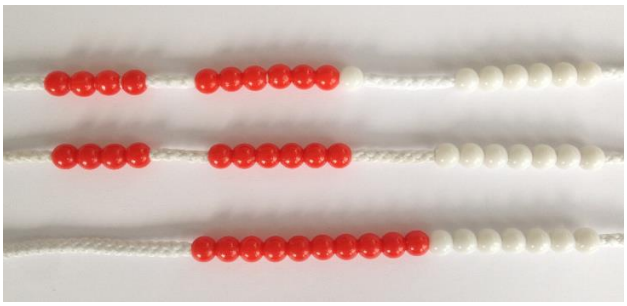
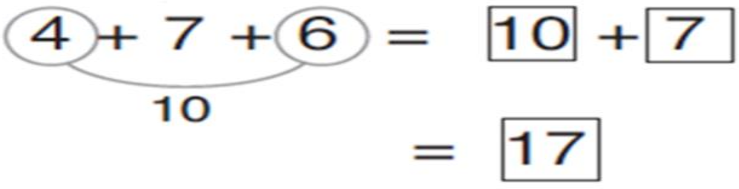
- Read, write and interpret mathematical statements involving addition (+), subtraction (−) and equal (=) signs.
- Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems, such as 7 = □ − 9.
- Solve problems with addition and subtraction:
    - Using concrete objects and pictorial representations, including those involving numbers, quantities and measures
    - Applying their increasing knowledge of mental and written methods

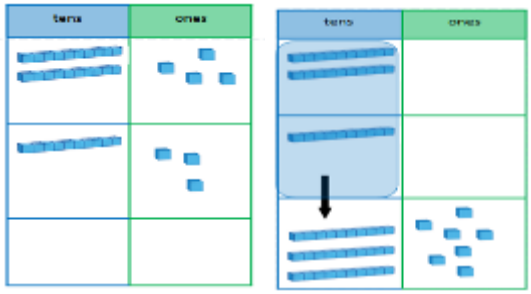
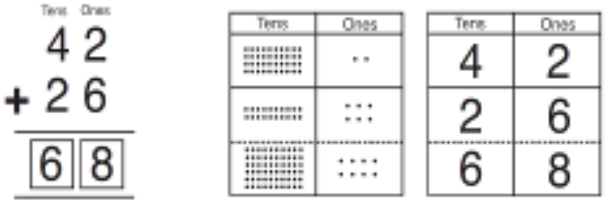
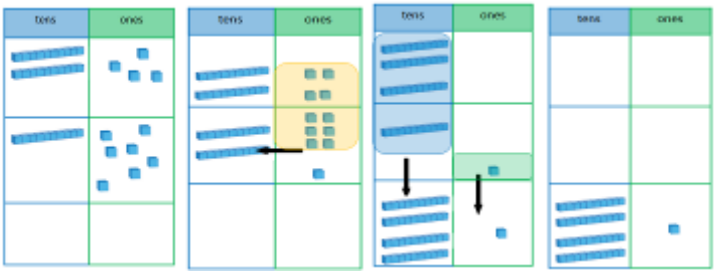
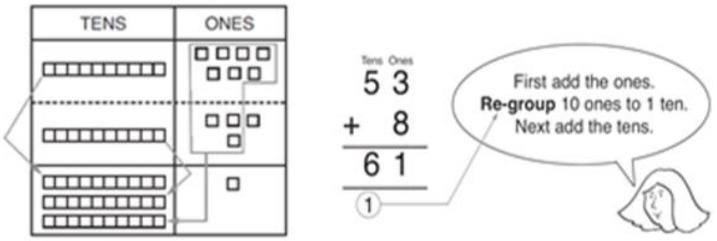
MATHEMATICS MASTERY

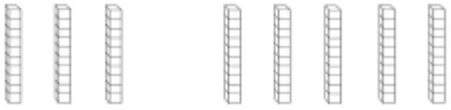
Addition

Strategy & guidance	CPA
Joining two groups and then recounting all objects using one-to-one correspondence	3 + 4 = 7   5 + 3 = 8
Counting on *Single digit number from a single digit number.* *Single digit number from a 2-digit number.*	8 + 1 = 9   17 = 12 + 5  
Part-part-whole *Teach both addition and subtraction alongside each other, as the pupils will use this model to identify the link between them.* *Pupils could place ten on top of the whole as well as writing it down. The parts could also be written in alongside the concrete representation.*	  10 = 6 + 4 10 − 6 = 4 10 − 4 = 6 10 = 4 + 6

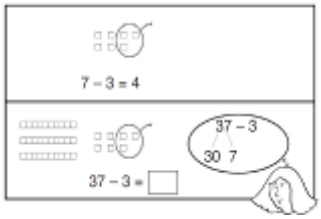
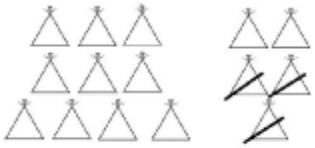
Regrouping ones to make ten *(This is an essential skill that will support the make ten strategy and column addition.)* *The colours of the beads on the bead string make it clear how many more need to be added to make ten.*	  3 + 9 = 12   
'Make ten' strategy *Pupils should be encouraged to start at the bigger number and use the smaller number to make ten.* *The colours of the beads on the bead string make it clear how many more need to be added to make ten.* *Also, the empty spaces on the ten frame make it clear how many more are needed to make ten.*	6 + 5 = 11  4 + 9 = 13  

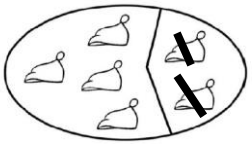
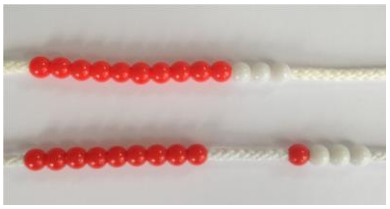
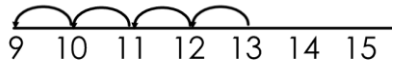
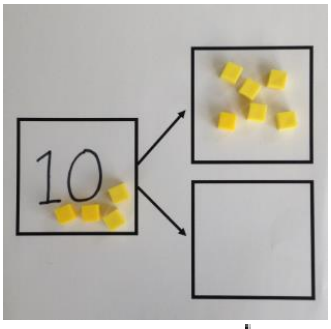
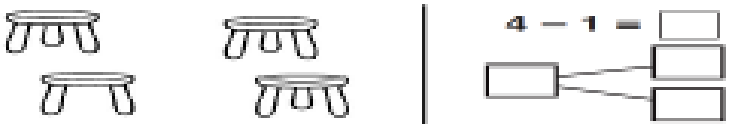
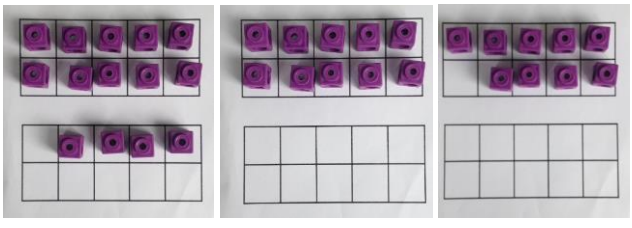
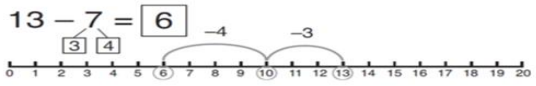
Adding 1, 2, 3 more *Here the emphasis should be on the language rather than the strategy. As pupils are using the beadstring, ensure that they are explaining using language such as;* *'1 more than 5 is equal to 6.'* *'2 more than 5 is 7.'* *'8 is 3 more than 5.'*	1 more than 5 5 + 1 = 6  2 more than 5 5 + 2 = 7   
Adding three single digit numbers (make ten first) *Pupils may need to try different combinations before they find the two numbers that make 10.* *The first bead string shows 4, 7 and 6. The colours of the bead string show that it makes more than ten.* *The second bead string shows 4, 6 and then 7.* *The final bead string shows how they have now been put together to find the total.*	 

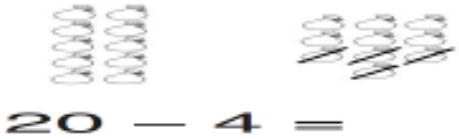
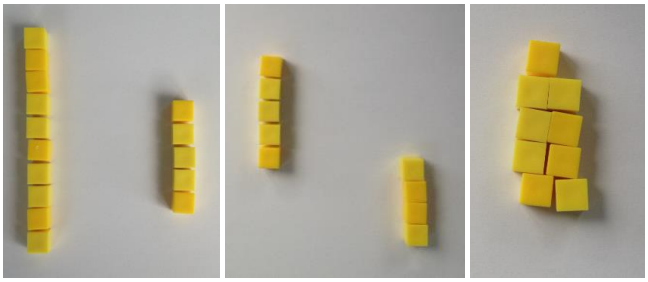
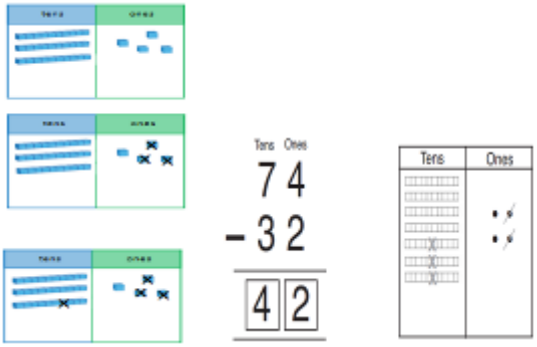
Column method for addition, no regrouping *Place value grids and Dienes blocks should be used as shown in the diagram before moving onto the pictorial representations. Dienes blocks should always be available, as the main focus in Year 1 is the concept of place value rather than mastering the procedure.* *See additional guidance on unit pages for extra guidance on this strategy.*	24 + 13 = 37  
Column method for addition, regrouping *Dienes blocks and place value grids should be used as shown in the diagrams. Even when working pictorially, pupils should have access to Dienes blocks.* *See additional guidance on unit pages for extra guidance on this strategy.*	 17 + 14 = 31 

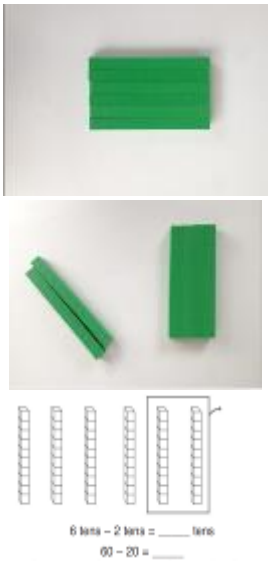
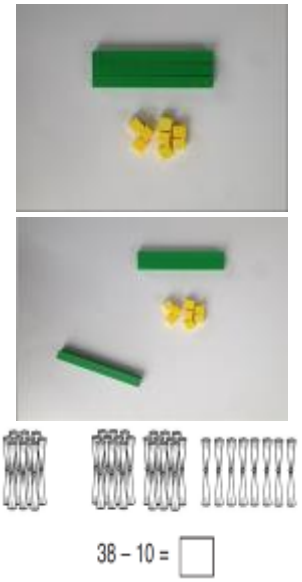
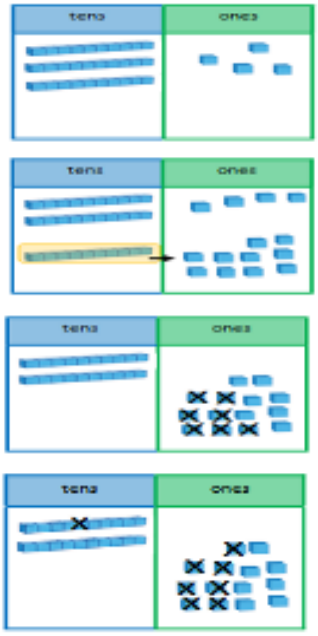
Adding multiples of ten *Using the vocabulary of 1 ten, 2 tens, 3 tens etc. alongside 10, 20, 30 is important, as pupils need to understand that it is a* ***ten*** *and not a one that is being added.*	50 = 30 + 20   3 tens + 5 tens = _____ tens 30 + 50 = _____  36 + 40 = ☐

Subtraction

Strategy & guidance	**CPA**
Taking away from the ones *When this is first introduced, the concrete representation should be based upon the diagram. Real objects should be placed on top of the images as one-to-one correspondence, progressing to representing the group of ten with a tens rod and ones with ones cubes.*	  15 – 3 = 12  6 – 2 = 4  28 - 4 =

Counting back *Single digit number from a single-digit number* *Single digit number from a 2 digit number*	  4 = 6 − 2   13 − 4 = 9
Part-part-whole *Teach both addition and subtraction alongside each other, as the pupils will use this model to identify the link between them.* *Pupils start with ten cubes placed on the whole.* *They then remove what is being taken away from the whole and place it on one of the parts.* *The remaining cubes are the other part and also the answer. These can be moved into the second part space.*	10 − 6 = 4  
Make ten strategy *single digit number from a 2-digit number* *Pupils identify how many need to be taken away to make ten first. Then they take away the rest to reach the answer.*	14 – 5 = 9  

Regroup a ten into 10 ones *After the initial introduction, the Dienes blocks should be placed on a place value chart to support place value understanding. This will then support pupils when using the column method.*	  20 — 4 =
Taking away from the tens *Pupils should begin to identify which equations require taking away from the tens and which from the ones.*	9 = 15− 6   16 — 8 = 8
Column method without regrouping *See additional guidance on unit pages to support with this strategy.*	34 − 13 = 21  Tens Ones 7 4 − 3 2 4 2 Tens Ones

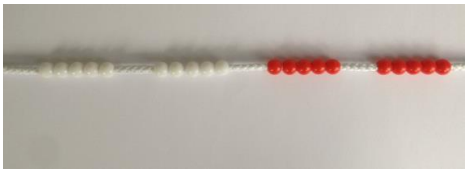
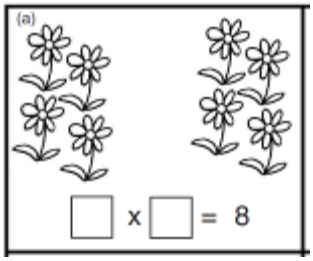
Subtracting multiples of ten *Using the vocabulary of 1 ten, 2 tens, 3 tens etc. alongside 10, 20, 30 is important as pupils need to understand that it is a **ten** not a one that is being taken away.*	40 = 60 – 20  38 – 10 = 28 
Column method with regrouping *This example shows how pupils should work practically when being introduced to this strategy.* *See additional guidance on unit pages to support with this strategy.*	 34 – 17 = 17

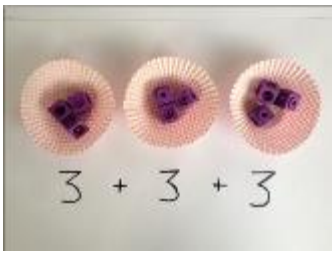
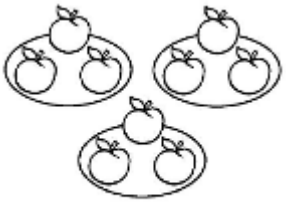
National Curriculum objectives linked to multiplication and division

These objectives are explicitly covered through the strategies outlined in this document:

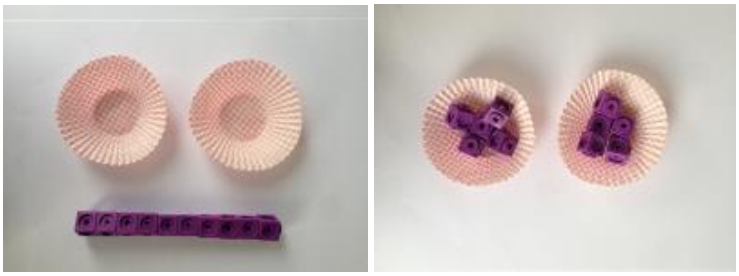
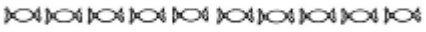
- Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Multiplication

Strategy & guidance	CPA
Skip counting in multiples of 2, 5, 10 from zero *The representation for the amount of groups supports pupils' understanding of the written equation. So two groups of 2 are 2, 4. Or five groups of 2 are 2, 4, 6, 8, 10.* *Count the groups as pupils are skip counting.* *Number lines can be used in the same way as the bead string.* *Pupils can use their fingers as they are skip counting.*	 4 × 5 = 20  2 × 4 = 8
Making equal groups and counting the total *How this would be represented as an equation will vary. This could be 2 × 4 or 4 × 2. The importance should be placed on the vocabulary used alongside the equation. So this picture could represent 2 groups of 4 or 4 twice.*	  Draw  to show 2 x 3 = 6

Solve multiplications using repeated addition	  How many apples are there altogether? 3 + 3 + 3 = 9

Division

Strategy & guidance	**CPA**
Sharing objects into groups *Pupils should become familiar with division equations through working practically.* *The division symbol is not formally taught at this stage.*	10 ÷ 2 = 5  There are 10 sweets. Ring groups of 2.  There are _____ groups of 2. Draw an equal number of apples for each basket.  There are five apples in each basket.

MATHEMATICS MASTERY

Progression in calculations

Year 2

AN ark PROGRAMME

National Curriculum objectives linked to addition and subtraction

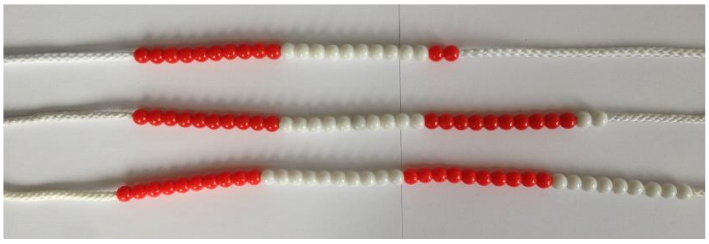
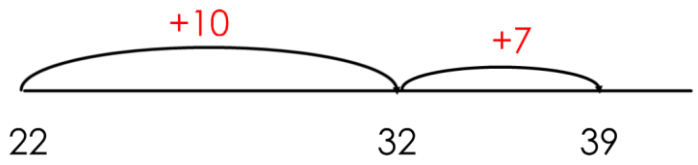
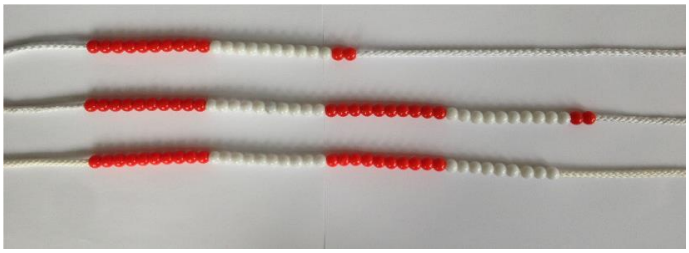
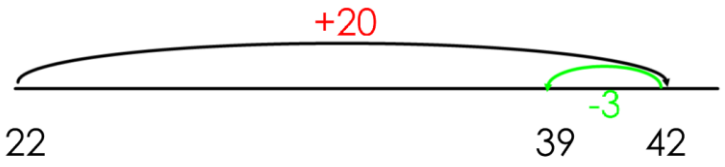
These objectives are explicitly covered through the strategies outlined in this document:

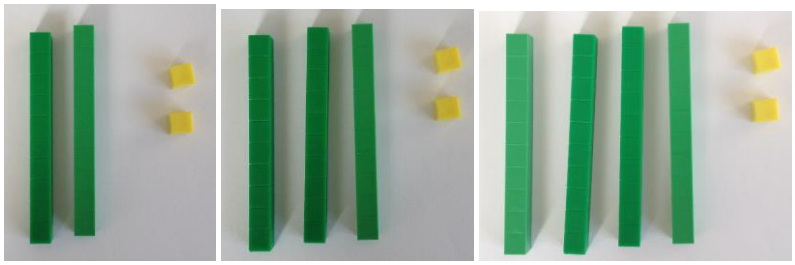
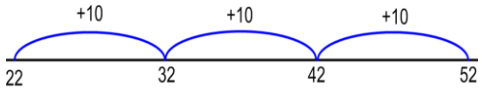
- Add and subtract numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and ones; a two-digit number and tens; 2 two-digit numbers; adding three one-digit numbers.
- Add and subtract numbers mentally, including: a three-digit number and ones; a three-digit number and tens; a three-digit number and hundreds (Year 3).
- Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100.
- Find 10 or 100 more or less than a given number (Year 3).
- Show that addition of two numbers can be done in any order (commutative) but subtraction of one number from another cannot.
- Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.
- Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction (Year 3).

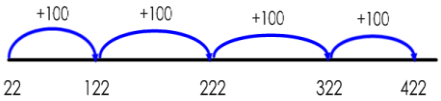
The following objectives should be planned for lessons where new strategies are being introduced and developed:

- Solve problems with addition and subtraction: using concrete objects and pictorial representations, including those involving numbers, quantities and measures; apply increasing knowledge of mental and written methods.
- Solve problems, including missing number problems, using number facts, place value and more complex addition and subtraction.

Addition

Strategy & guidance	CPA
Partitioning one number, then adding tens and ones *Pupils can choose themselves which of the numbers they wish to partition. Pupils will begin to see when this method is more efficient than adding tens and taking away the extra ones, as shown.*	  22 + 17 = 39
Rounding one number, then adding the tens and taking away extra ones *Pupils will develop a sense of efficiency with this method, beginning to see when rounding and adjusting is more efficient than adding tens and then ones.*	  22 + 17 = 39

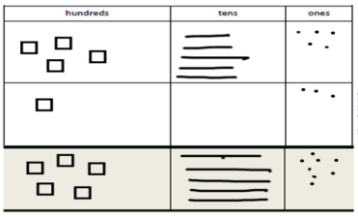
<table>
<tr>
<td>Counting on in tens and hundreds</td>
<td>





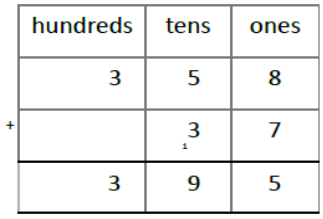
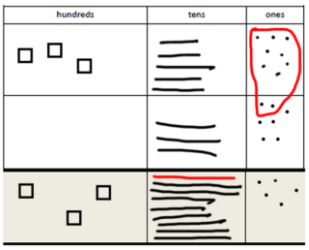
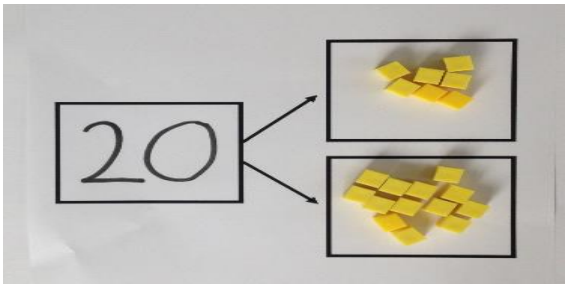
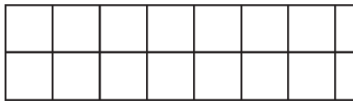
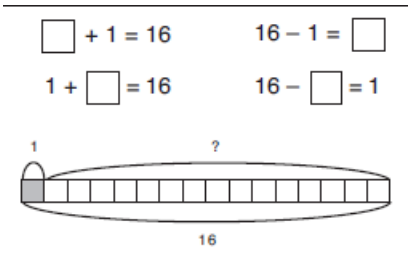
</td>
</tr>
<tr>
<td>Column method without regrouping

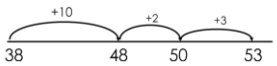
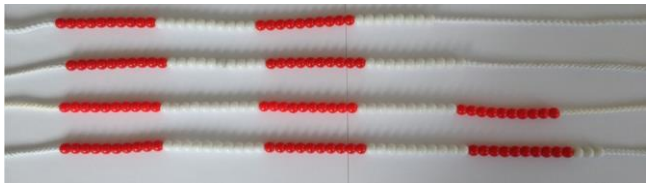
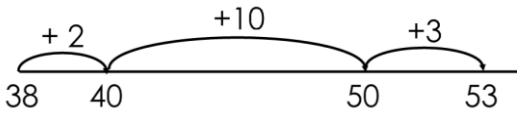
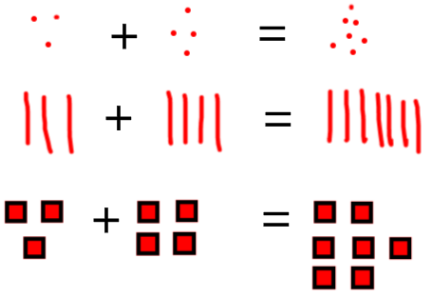
Dienes blocks should be used alongside the pictorial representations; they can be placed on the place value grid before pupils make pictorial representations.

As in Year 1, the focus for the column method is to develop a strong understanding of place value.

Please also see additional guidance on unit pages for extra guidance on this strategy.</td>
<td>


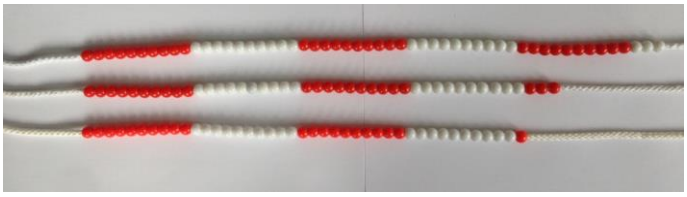
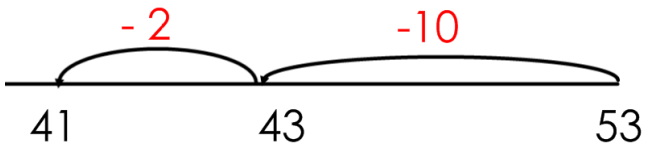
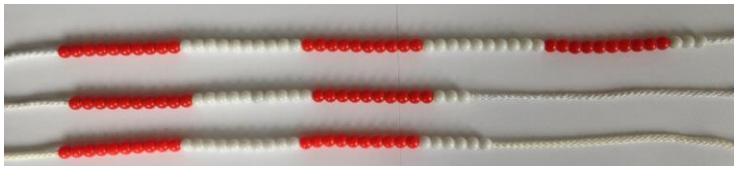
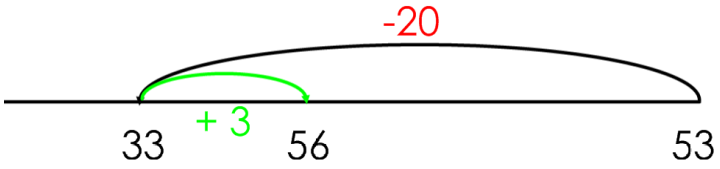
<table>
<tr><th>hundreds</th><th>tens</th><th>ones</th></tr>
<tr><td>4</td><td>5</td><td>5</td></tr>
<tr><td>1</td><td>0</td><td>3</td></tr>
<tr><td>5</td><td>5</td><td>8</td></tr>
</table>
</td>
</tr>
</table>

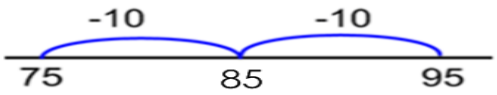
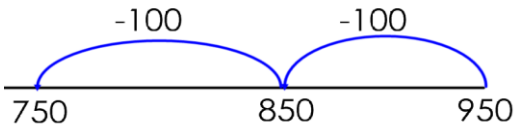
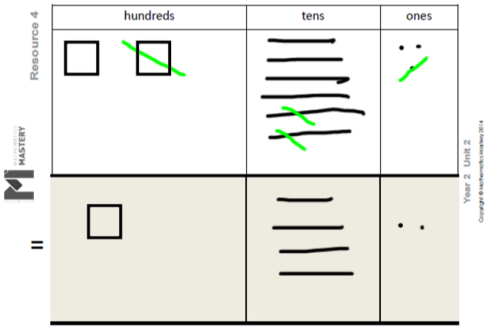
Column method with regrouping *Dienes blocks should be used alongside the pictorial representations; they can be placed on the place value grid before pupils make pictorial representations.* *As in Year 1, the focus for the column method is to develop a strong understanding of place value.* *See additional guidance on unit pages for extra guidance on this strategy.*	 
Part-part-whole *Pupils explore the different ways of making 20. They can do this with all numbers using the same representations.*	   

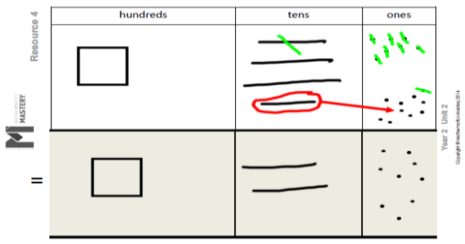
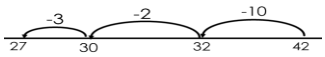
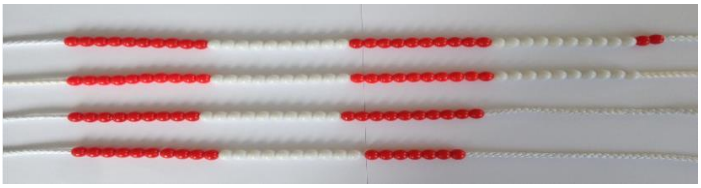
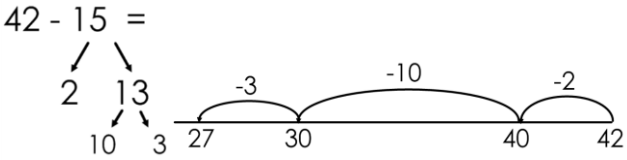
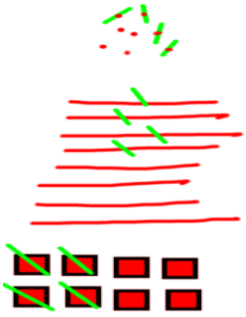
Make ten strategy  *How pupils choose to apply this strategy is up to them; however, the focus should always be on efficiency.*	 38 + 15 = 2 13 10 3 
Using known facts *Dienes blocks should be used alongside pictorial and abstract representations when introducing this strategy.*	 3 + 4 = 7 *leads to* 30 + 40 = 70 *leads to* 300 + 400 = 700

MATHEMATICS MASTERY

Subtraction

Strategy & guidance	CPA
Subtracting tens and ones *Pupils must be taught to partition the second number for this strategy.* *Pupils will begin to see when this method is more efficient than subtracting tens and adding the extra ones, as shown.*	  53 – 12 = 41
Subtracting tens and adding extra ones *Pupils must be taught to round the number that is being subtracted.* *Pupils will develop a sense of efficiency with this method, beginning to identify when this method is more efficient than subtracting tens and then ones.*	  53 – 17 = 36

Counting back in multiples of ten and one hundred	  
Column method without regrouping *As in Year 1, the focus for the column method is to develop a strong understanding of place value and pupils should always be using concrete manipulatives alongside the pictorial.* *Please also see additional guidance on unit pages for extra guidance on this strategy.*	 263 – 121= 142

Column method with regrouping *As in Year 1, the focus for the column method is to develop a strong understanding of place value and pupils should always be using concrete manipulatives alongside the pictorial.* *See additional guidance on unit pages for extra guidance on this strategy.*	 147 – 18 = 129
Bridging through ten  *How pupils choose to apply this strategy is up to them. The focus should always be on efficiency.*	 
Using known number facts *Dienes blocks should be used alongside pictorial and abstract representations when introducing this strategy.*	 8 – 4 = 4 *leads to* 80 – 40 = 40 *leads to* 800 – 400 = 400

National Curriculum objectives linked to multiplication and division

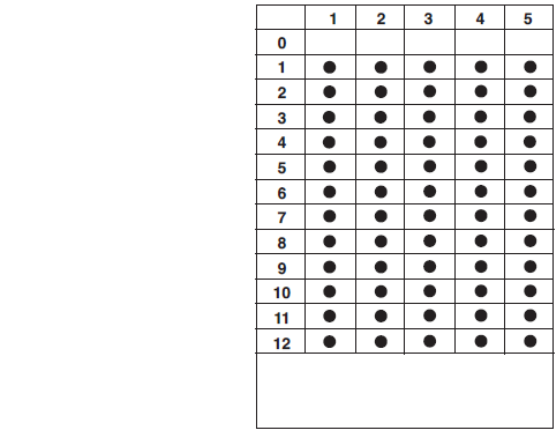
These objectives are explicitly covered through the strategies outlined in this document:

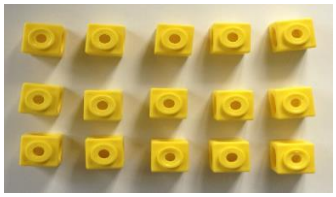
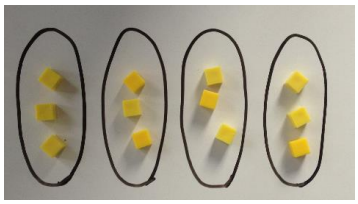
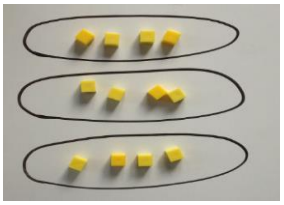
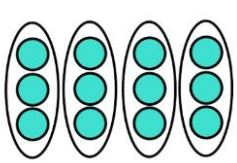
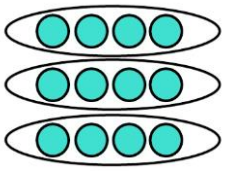
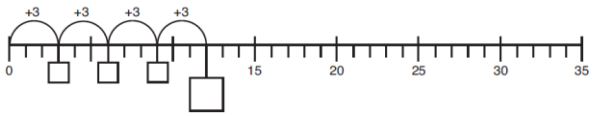
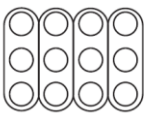
- Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers.
- Recall and use multiplication and division facts for the 3 and 4 multiplication tables (Year 3).
- Show that multiplication of two numbers can be done in any order (commutative) but division of one number by another cannot.

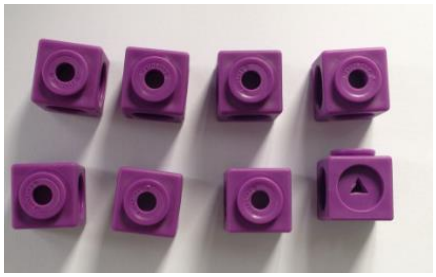
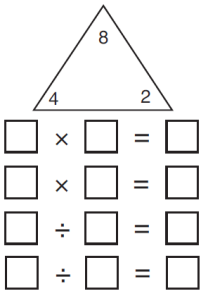
The following objectives should be planned for lessons where new strategies are being introduced and developed:

- Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equal (=) signs.
- Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods and multiplication and division facts, including problems in context.

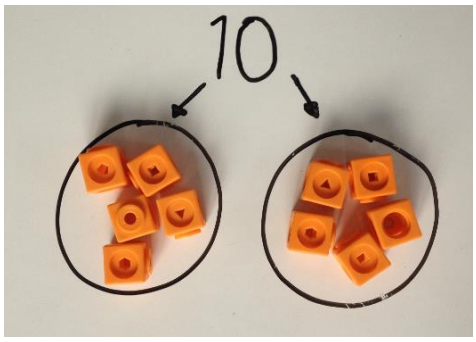
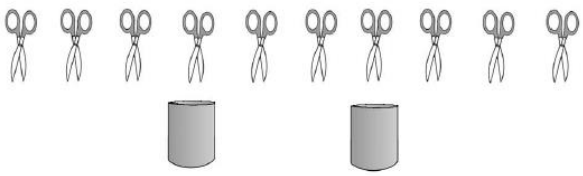
Multiplication

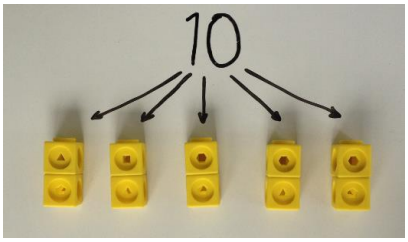
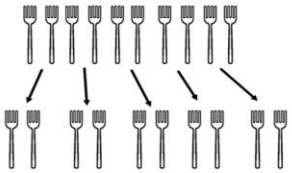
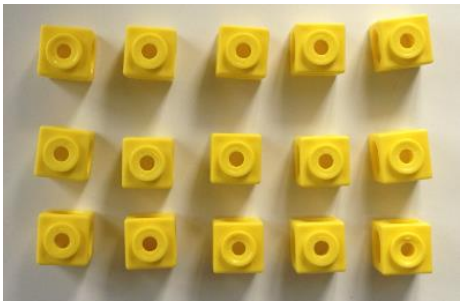
Strategy & guidance	
Skip counting in multiples of 2, 3, 4, 5, 10 from 0 *Pupils can use their fingers as they are skip counting, to develop an understanding of 'groups of'.* *Dotted paper is used to create a visual representation for the different multiplication facts. Each multiplication table has its own template, which is provided during taught units.*	MATHEMATICS MASTERY Resource 2 Dotted paper (multiplication table of 5)  

Multiplication is commutative *Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.*	 3 x 5 = ☐ 5 x 3 = ☐      12 = 3 × 4 12 = 4 × 3
Multiplication as repeated addition *Pupils will apply skip counting to help find the totals of these repeated additions.*	 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = ☐     4 × 3 = ☐

Using the inverse *This should be taught alongside division, so pupils learn how they work alongside each other.*	 

Division

Strategy & guidance	
Division as sharing *Here, division is shown as sharing. If we have ten pairs of scissors and we share them between two pots, there will be 5 pairs of scissors in each pot.*	 10 ÷ 2 = 5 

Division as grouping *Here, division is shown as grouping. If we have ten forks and we put them into groups of two, there are 5 groups.*	10 ÷ 2 = 5  
Using the inverse *This should be taught alongside multiplication so that pupils learn how they work alongside each other.*	 15 ÷ 5 = 3 15 ÷ 3 = 5